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THREE-DIMENSIONAL SUPERGRAVITY AND THE COSMOLOGICAL CONSTANT

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ABSTRACT

Witten has argued that in 2+1 dimensions local supersymmetry can ensure the vanishing of the cosmological constant without requiring the equality of boson and fermion masses. We find that this mechanism is implemented in a novel fashion in the (2+1)-dimensional supersymmetric abelian Higgs model coupled to supergravity. The vortex solitons are annihilated by half of the supersymmetry transformations. The covariantly constant spinors required to define these supersymmetries exist by virtue of a surprising cancellation between the Aharonov-Bohm phase and the phase associated with the holonomy of the spin connection. However the other half of the supersymmetry transformations, whose actions ordinarily generate the soliton supermultiplet, are not well-defined and boson-fermion degeneracy is consequently absent in the soliton spectrum.

The cosmological constant problem is surely one of the most vexing problems in all of physics[★]. Supersymmetry has something to say about the problem, but it is not clear if it makes matters better or worse. In some theories (*e.g.* string theory) the cosmological constant can naturally vanish before supersymmetry breaking. However after supersymmetry breaking it is typically non-zero, and cannot be made to vanish even by fine-tuning bare parameters.

Some time ago [2] Witten made a striking observation concerning the relationship between local supersymmetry and the cosmological constant in $2 + 1$ dimensions. The vacuum can have exactly zero cosmological constant because of local supersymmetry, yet the excited states may not be in degenerate bose-fermi pairs. This is because supercharges – whose existence ordinarily implies bose-fermi degeneracies – are ill-defined in the conical geometries arising in non-zero energy states in $2 + 1$ dimensions [3]. Thus in $2 + 1$ we can have our cake and eat it, too. It would certainly be wonderful if a $3 + 1$ dimensional theory in which supersymmetry implied zero cosmological constant but not the unwanted degenerate bose-fermi pairs could be found! Unfortunately there have been no suggestions of how to implement this idea in $3 + 1$.

In this paper we investigate this mechanism as it applies to the solitons of the $N = 2$ supersymmetric abelian Higgs model coupled to supergravity in $2+1$ dimensions. Before coupling to supergravity this theory has a supermultiplet of Nielsen-Olesen vortex solitons [4] of mass M . The solitons are annihilated by half of the supersymmetry transformations. The action of the other, broken, half generates fermionic Nambu-Goldstone zero modes. Quantization of these zero modes leads to a supermultiplet of degenerate bosonic and fermionic soliton states. We shall find that the unbroken supersymmetries ingeniously survive the coupling to supergravity, despite the existence of a conical geometry. This is possible because in the locally supersymmetric theory the supersymmetry transformation parameter becomes charged. The geometric phase associated to the conical geometry is

★ An excellent review of the cosmological constant problem can be found in ref. [1]

then cancelled by an Aharonov-Bohm phase. On the other hand the phases add rather than cancel for the would-be broken supersymmetry generators. There are accordingly no normalizable fermion zero modes, and the bose-fermi degeneracy is split.

Curiously this model contains fermionic particles with potentially fractional charges $v^2 g/2M_p$, where g is the Higgs charge, the constant v is the magnitude of the Higgs vacuum expectation value, v^2 is the coefficient of the Fayet-Iliopoulos D -term and M_p is the Planck Mass. If topologically non-trivial gauge connections[†] are allowed, and g is an integer multiple p of the fundamental electric charge, then the coupling to supergravity is only consistent if v is quantized according to $pv^2 = 2qM_p$, where q is an integer. A similar quantization condition[‡] applies to the four-dimensional Abelian Higgs model coupled to supergravity but does not appear to have been previously noticed.

We first review the solitons of the $(2 + 1)$ -dimensional abelian Higgs model with $N = 2$ global supersymmetry. The Lagrangian is [6,7,8]

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \mathcal{D}_\mu\phi\mathcal{D}^\mu\phi^* - \frac{1}{2}\partial_\mu N\partial^\mu N - g^2 N^2\phi\phi^* - \frac{g^2}{2}D^2(\phi) \\ & - i\bar{\chi}\not{D}\chi - i\bar{\lambda}\not{\partial}\lambda + i\sqrt{2}g(\bar{\chi}\lambda\phi - \bar{\lambda}\chi\phi^*) - gN\bar{\chi}\chi, \end{aligned} \quad (1)$$

where

$$D(\phi) = \phi^*\phi - v^2. \quad (2)$$

Here g is the gauge coupling (with dimensions of $(\text{mass})^{1/2}$), N is a neutral real scalar, ϕ is a complex charged scalar, χ (λ) is a complex charged (neutral) two-component spinor, $\mathcal{D}_\mu = \partial_\mu - igA_\mu$ is the covariant derivative when acting on ϕ

[†] *i.e.* connections for which the integral of the field strength F over a closed two surface is non-zero.

[‡] This quantization condition is related to the non-renormalization theorems for the Fayet-Iliopoulos D -term discussed in the context of string theory [5]. The situation encountered in string theory is somewhat different in that a non-linear $U(1)$ transformation law for the axion (associated with anomaly cancellation) cancels the shift in $U(1)$ fermion charges.

and $\bar{\chi} = \chi^\dagger \gamma^0$, *etc.*. The metric tensor $\eta_{\mu\nu}$ has the signature $(-, +, +)$. The γ matrices can be represented by $\gamma^0 = \sigma^3$, $\gamma^1 = i\sigma^2$ and $\gamma^2 = i\sigma^1$ and they satisfy the relation $\gamma^\mu \gamma^\nu = -\eta^{\mu\nu} - i\varepsilon^{\mu\nu\lambda} \gamma_\lambda$.

This theory is invariant under $N = 2$ supersymmetry transformations:

$$\begin{aligned}
\delta_\varepsilon \chi &= i\sqrt{2}\gamma^\mu \mathcal{D}_\mu \phi \varepsilon - \sqrt{2}gN\phi \varepsilon, \\
\delta_\varepsilon \lambda &= F_{\mu\nu} \gamma^{\mu\nu} \varepsilon - igD(\phi)\varepsilon - \partial_\mu N \gamma^\mu \varepsilon, \\
\delta_\varepsilon A_\mu &= i(\bar{\varepsilon} \gamma_\mu \lambda - \bar{\lambda} \gamma_\mu \varepsilon), \\
\delta_\varepsilon \phi &= \sqrt{2}\bar{\varepsilon} \chi, \\
\delta_\varepsilon N &= i(\bar{\lambda} \varepsilon - \bar{\varepsilon} \lambda).
\end{aligned} \tag{3}$$

Here the parameter ε is a complex anticommuting spinor, $\gamma^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$.

The soliton is given by the static, vortex field configuration obeying the first order differential equations

$$F \equiv \varepsilon^{z\bar{z}} F_{z\bar{z}} = gD(\phi) \quad \text{and} \quad \mathcal{D}_z \phi = 0, \tag{4}$$

where (z, \bar{z}) are complex spatial coordinates, $\varepsilon^{z\bar{z}} = -\varepsilon^{\bar{z}z} = -i\eta^{z\bar{z}} = -2i$ in flat space and

$$F_{0z} = F_{0\bar{z}} = N = 0. \tag{5}$$

Antivortices with $F = -gD(\phi)$ and $\mathcal{D}_z \phi^* = 0$ also exist.

The solutions of the first order differential equations (4) are labeled by the magnetic flux

$$\int F d^2 z = -\frac{4\pi}{g} n \quad \text{with} \quad n \in \mathbb{Z}_+, \tag{6}$$

while antivortices satisfy equation (6) with $n \in \mathbb{Z}_-$. In the following we take n positive.

The soliton solutions are known to exist but they cannot be found analytically by solving equation (4). However what is important for our considerations is that all local gauge invariant quantities fall to zero exponentially outside of a core region of characteristic size $1/v^2$, because there are no massless propagating fields.

The solution (4) breaks half of the supersymmetries of the theory. To see this decompose ε into spinors ε_+ and ε_- of definite spatial chiralities:

$$\begin{aligned}\gamma^{\bar{z}}\varepsilon_+ &= 0, \\ \gamma^z\varepsilon_- &= 0.\end{aligned}\tag{7}$$

Then it is easy to see that the transformations generated by ε_+ are unbroken

$$\delta_{\varepsilon_+}\chi = \delta_{\varepsilon_+}\lambda = 0,\tag{8}$$

The transformations generated by ε_- are broken

$$\begin{aligned}\delta_{\varepsilon_-}\chi &= i\sqrt{2}\gamma^{\bar{z}}(\mathcal{D}_{\bar{z}}\phi)\varepsilon_- \neq 0, \\ \delta_{\varepsilon_-}\lambda &= -2igD(\phi)\varepsilon_- \neq 0,\end{aligned}\tag{9}$$

except when we are in the vacuum where $D(\phi) = 0$, *i.e.* $|\phi| = v$. By virtue of Goldstone's theorem we expect a massless excitation for every broken symmetry. Indeed (9) are the Nambu-Goldstone zero modes in the soliton background. They are normalizable because of the exponential falloff of $D(\phi)$ and $\mathcal{D}_{\bar{z}}\phi$.

The solitonic spectrum is obtained by quantizing these zero modes. The operator b_0 which creates a fermion zero mode obeys:

$$\begin{aligned}\{b_0^*, b_0\} &= 1, \\ \{b_0, b_0\} &= \{b_0^*, b_0^*\} = 0\end{aligned}\tag{10}$$

and of course carries no energy. The soliton groundstate for $n = 1$ is then a representation of (10) corresponding to a massive $(0, 1/2)$ supermultiplet.

For the model with global supersymmetry, we can conclude that the cosmological constant vanishes because all the supersymmetries are unbroken in the vacuum [9]. However, there is a phenomenologically undesirable degeneracy between bosons and fermions. This situation will change for the model with local supersymmetry.

The locally supersymmetric action in $2 + 1$ dimensions does not seem to appear explicitly in the literature but may be obtained by reduction of the four-dimensional $N = 1$ abelian Higgs model coupled to supergravity [10,11]. The resulting Lagrangian is

$$\mathcal{L} = \frac{M_p}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \mathcal{D}_\mu\phi\mathcal{D}^\mu\phi^* - \frac{1}{2}\mathcal{D}_\mu N\mathcal{D}^\mu N - g^2 N^2\phi^*\phi - \frac{g^2}{2}D(\phi)^2 + (\text{fermi}). \quad (11)$$

where \mathcal{D}_μ is the covariant derivative with respect to gravity and the $U(1)$ gauge group. The full Lagrangian is invariant under local supersymmetry transformations and under the $U(1)$ gauge transformations

$$\begin{aligned} \delta_\alpha\phi &= i\alpha\phi, \\ \delta_\alpha A_\mu &= g^{-1}\partial_\mu\alpha, \\ \delta_\alpha\chi &= i\alpha\chi + \frac{iv^2\alpha}{2M_p}\chi, \\ \delta_\alpha\lambda &= \frac{iv^2\alpha}{2M_p}\lambda, \\ \delta_\alpha\psi_\mu &= \frac{iv^2\alpha}{2M_p}\psi_\mu. \end{aligned} \quad (12)$$

From this formula we observe that the gravitino is charged while the charges of other fermions are shifted when supergravity is coupled. These charges have their origin in the Kähler invariance of the Lagrangian [12]. If topologically non-trivial connections are allowed^{*} and g is an integer multiple p of the minimal electric charge, charge quantization implies $pv^2 = 2qM_p$ where q is an integer.

^{*} The vortex solution on its own is not incompatible with fractional charge because all of spacetime can be covered with one patch. Problems with fractionally charged objects arise when there are non trivial transitions between neighboring patches.

The relevant part of the supersymmetry transformations becomes^{*}

$$\begin{aligned}\delta_\varepsilon \chi &= i\sqrt{2}\gamma^\mu \mathcal{D}_\mu \phi \varepsilon, \\ \delta_\varepsilon \lambda &= F_{\mu\nu} \gamma^{\mu\nu} \varepsilon - igD(\phi)\varepsilon, \\ \delta_\varepsilon \psi_\mu &= \mathcal{D}_\mu^- \varepsilon.\end{aligned}\tag{13}$$

where we have defined the Kähler-covariant derivative

$$\mathcal{D}_\nu^\pm = \mathcal{D}_\nu \pm \frac{J_\nu}{4M_p} \quad \text{and} \quad J_\nu = \phi^* \mathcal{D}_\nu \phi - \phi \mathcal{D}_\nu \phi^*.\tag{14}$$

The supersymmetry parameter ε is charged and transforms as:

$$\delta_\alpha \varepsilon = \frac{iv^2 \alpha}{2M_p} \varepsilon.\tag{15}$$

The bosonic equations of motion that follow from the Lagrangian are

$$\begin{aligned}M_p G_{\mu\nu} = T_{\mu\nu} &\equiv -\frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta} - g_{\mu\nu}\mathcal{D}_\alpha \phi \mathcal{D}^\alpha \phi^* + \mathcal{D}_\mu \phi \mathcal{D}_\nu \phi^* \\ &\quad + \mathcal{D}_\mu \phi^* \mathcal{D}_\nu \phi - \frac{1}{2}g_{\mu\nu}\mathcal{D}_\alpha N \mathcal{D}^\alpha N + \mathcal{D}_\nu N \mathcal{D}_\mu N - \frac{g^2}{2}g_{\mu\nu}D(\phi)^2, \\ \mathcal{D}_\mu \mathcal{D}^\mu \phi &= g^2 \phi D(\phi), \\ \mathcal{D}^\mu F_{\mu\nu} &= igJ_\nu.\end{aligned}\tag{16}$$

As in flat space we will look for static solutions that satisfy $F_{0z} = F_{0\bar{z}} = N = 0$.

For such solutions the line element can be put in the form

$$ds^2 = -dt^2 + e^{2\rho} dz d\bar{z},\tag{17}$$

and the t - t component of Einsteins equation takes the form

$$M_p R_{z\bar{z}} = -\frac{1}{2}e^{2\rho}v^2 Fg + \frac{1}{2}(\mathcal{D}_z J_{\bar{z}} - \mathcal{D}_{\bar{z}} J_z) + \frac{1}{4}e^{2\rho}(F - gD(\phi))^2 + 2\mathcal{D}_z \phi \mathcal{D}_{\bar{z}} \phi^*.\tag{18}$$

It is then easy to see that the matter equations of motion are solved by the Landau-

* This corrects a misprint in ref. [11].

Ginzburg vortices

$$F = gD(\phi) \quad \text{and} \quad \mathcal{D}_z \phi = 0. \quad (19)$$

Using (19), equation (18) reduces to a linear relation for ρ

$$M_p R_{z\bar{z}} = -2M_p \partial_z \partial_{\bar{z}} \rho = iv^2 g F_{z\bar{z}} + \frac{1}{2} (\mathcal{D}_z J_{\bar{z}} - \mathcal{D}_{\bar{z}} J_z). \quad (20)$$

Asymptotically the right hand side of equation (18) vanishes, and the geometry is therefore locally flat. The metric takes the form

$$ds^2 \simeq -dt^2 + \frac{dzd\bar{z}}{(z\bar{z})^{M/M_p}} \quad \text{as} \quad r^2 = z\bar{z} \rightarrow \infty, \quad (21)$$

where

$$M = v^2 n \quad (22)$$

(with $n > 0$) is proportional to the soliton mass. This corresponds to a cone with deficit angle $2\pi M/M_p$. From equation (21) we observe that if $M > M_p$, the metric becomes singular [3]. For the marginal case $M = M_p$ the space is asymptotic to a cylinder.

It is easy to see that as in the global case,

$$\delta_{\varepsilon_+} \chi = \delta_{\varepsilon_+} \lambda = 0, \quad (23)$$

for any positive chirality spinor ε_+ . Half of the supersymmetries are unbroken if we can find a specific ε_+ with the additional property $\delta_{\varepsilon_+} \psi_\mu = 0$. If ε_+ is time-independent, $\delta_{\varepsilon_+} \psi_0$ will trivially vanish. The conditions

$$\begin{aligned} \delta_{\varepsilon_+} \psi_z &= \mathcal{D}_z^- \varepsilon_+ = \partial_z \varepsilon_+ - \frac{J_z}{4M_p} \varepsilon_+ - i \frac{v^2 g}{2M_p} A_z \varepsilon_+ = 0, \\ \delta_{\varepsilon_+} \psi_{\bar{z}} &= \mathcal{D}_{\bar{z}}^- \varepsilon_+ = \partial_{\bar{z}} \varepsilon_+ - \partial_{\bar{z}} \rho \varepsilon_+ - \frac{J_{\bar{z}}}{4M_p} \varepsilon_+ - i \frac{v^2 g}{2M_p} A_{\bar{z}} \varepsilon_+ = 0, \end{aligned} \quad (24)$$

are differential equations for ε_+ . Solutions will exist if the integrability condition

$$[\mathcal{D}_z^-, \mathcal{D}_{\bar{z}}^-] \varepsilon_+ = 0 \quad (25)$$

is satisfied. This condition is indeed equivalent to the constraint equation (20), and we conclude that half of the supersymmetries are unbroken.

This result may come as a surprise since covariantly constant spinors do not usually exist in asymptotically conical spaces: a phase is acquired in parallel transport about a circle at infinity. However in this case the phase is cancelled by an Aharonov-Bohm phase which arises because the gravitino has charge! To see this explicitly note that asymptotically as $r \rightarrow \infty$

$$\partial_z \rho \rightarrow -\frac{M}{2M_p z}, \quad (A_z, A_{\bar{z}}) \rightarrow \frac{in}{2g} \left(\frac{1}{z}, -\frac{1}{\bar{z}} \right) \quad \text{and} \quad (J_z, J_{\bar{z}}) \rightarrow (0, 0) \quad (26)$$

to leading order in $1/r$, where we have used (6), (21) and the relation

$$\int_{\Sigma} d^2 z \partial_{\bar{z}} f(z, \bar{z}) = -i \oint_{\partial \Sigma} dz f(z, \bar{z}). \quad (27)$$

Using $M = nv^2$, cancellations occur between the connections, and the covariant constancy conditions (24) reduce to

$$\begin{aligned} \mathcal{D}_z^- \varepsilon_+ &\rightarrow \partial_z \varepsilon_+ + \frac{M}{4M_p z} \varepsilon_+ = 0, \\ \mathcal{D}_{\bar{z}}^- \varepsilon_+ &\rightarrow \partial_{\bar{z}} \varepsilon_+ + \frac{M}{4M_p \bar{z}} \varepsilon_+ = 0. \end{aligned} \quad (28)$$

The solutions obey

$$\varepsilon_+ \rightarrow (z\bar{z})^{-M/4M_p} \varepsilon_0 \quad \text{and} \quad e^{-\rho} \varepsilon_+^\dagger \varepsilon_+ \rightarrow \text{constant} \quad (29)$$

as expected for a parameter which generates a nontrivial global supersymmetry transformation.

Stability of this solution follows from a Bogomol'nyi bound [13] relating the mass of a configuration to the magnetic flux:

$$M \geq v^2 |n|. \quad (30)$$

The above inequality is saturated if the configurations satisfy the first order differential equations. This bound can be derived using a variant of the methods of refs. [14,15]. Because of the infrared divergences, it appears necessary to work at large, but finite r . Define

$$\Delta(r) \equiv i \oint_{\mathcal{C}_r} dx^\alpha \bar{\eta} \mathcal{D}_\alpha^- \eta. \quad (31)$$

η here is an anticommuting spinor which transforms like ε and will be further constrained below. The integration contour \mathcal{C}_r is a curve of fixed r embedded in a spacelike slice Σ on which the metric asymptotically approaches the conical form (21). $\Delta(r)$ may be expressed as a volume integral over the portion Σ_r of Σ inside \mathcal{C}_r . One finds

$$\begin{aligned} \Delta(r) = \int_{\Sigma_r} d^2 \Sigma_\mu [i \varepsilon^{\mu\alpha\beta} \mathcal{D}_\alpha^+ \bar{\eta} \mathcal{D}_\beta^- \eta - \frac{1}{2} G^{\mu\alpha} \bar{\eta} \gamma_\alpha \eta + \frac{v^2 g}{4M_p} \varepsilon^{\mu\alpha\beta} F_{\alpha\beta} \bar{\eta} \eta \\ - \frac{i}{4M_p} \bar{\eta} \eta \varepsilon^{\mu\alpha\beta} \mathcal{D}_\alpha J_\beta]. \end{aligned} \quad (32)$$

It is always possible to find a coordinate system in which the metric takes the simple form (21) on Σ (though this can not be done throughout the entire spacetime for nonstatic geometries). With respect to such coordinates we further impose the condition

$$\gamma^{\bar{z}} \eta = \mathcal{D}_z^- \eta = 0, \quad (33)$$

so that η reduces to a single complex component which we denote η_+ . $\Delta(r)$ then

reduces to

$$\Delta(r) = \int d^2z [e^{-\rho} |\mathcal{D}_{\bar{z}}^- \eta_+|^2 + e^{\rho} \frac{1}{8M_p} |\eta|^2 (F - gD(\phi))^2 + e^{-\rho} \frac{1}{M_p} |\eta|^2 |\mathcal{D}_z \phi|^2], \quad (34)$$

where $|\eta|^2 = \eta_+^\dagger \eta_+$, *etc.* Evidently

$$\Delta(r) \geq 0. \quad (35)$$

The equality holds if and only if every term vanishes, which implies $M = v^2 n$. On the other hand the asymptotic behavior of η can be read off from (33) after inserting the asymptotic form (26) of the connection

$$\eta \sim r^{-v^2 n / 2M_p}. \quad (36)$$

It follows that as $r \rightarrow \infty$

$$\Delta(r) \sim (M - v^2 n) r^{(M - v^2 n) / 2M_p}. \quad (37)$$

Now suppose that M is less than $v^2 n$. Then $\Delta(r)$ is negative for large r . But this contradicts (35). One may also show that $M \geq -v^2 n$ for antivortices with $n < 0$ by considering spinors which obey $\gamma^z \eta = \mathcal{D}_{\bar{z}}^- \eta = 0$. We conclude that the bound (30) is valid, and is saturated by the stable solution (21).

One might expect that Nambu-Goldstone zero modes can be constructed from the broken supersymmetries. As in the global case

$$\begin{aligned} \delta_{\varepsilon_-} \chi &= i\sqrt{2} \gamma^{\bar{z}} (\mathcal{D}_{\bar{z}} \phi) \varepsilon_- \neq 0, \\ \delta_{\varepsilon_-} \lambda &= -2igD(\phi) \varepsilon_- \neq 0, \\ \delta_{\varepsilon_-} \psi_z &= \partial_z \varepsilon_- - \partial_z \rho \varepsilon_- - \frac{J_z}{4M_p} \varepsilon_- - i \frac{v^2 g}{2M_p} A_z \varepsilon_- \neq 0, \\ \delta_{\varepsilon_-} \psi_{\bar{z}} &= \partial_{\bar{z}} \varepsilon_- - \frac{J_{\bar{z}}}{4M_p} \varepsilon_- - i \frac{v^2 g}{2M_p} A_{\bar{z}} \varepsilon_- \neq 0, \end{aligned} \quad (38)$$

is a zero mode. However if ε_- has the asymptotic behavior (29) corresponding to a physical (*i.e.* not pure gauge) supersymmetry transformation, the zero mode (38)

is not normalizable. The norm of this mode has an infrared divergent contribution

$$\int d^2z (\delta_{\varepsilon_-} \psi_\mu)^* \delta_{\varepsilon_-} \psi_\mu e^{-\rho} \sim \int d^2z (z\bar{z})^{-1}. \quad (39)$$

Thus in the supergravity theory the would-be Nambu-Goldstone zero mode picks up a small, but long-range gravitino term which renders it non-normalizable, and so it does not enter in the construction of the physical Hilbert space[★].

While there are no physical fermion zero modes for a single soliton, there is a lowest-lying eigenmode. As M_p is made large, and $M/M_p \rightarrow 0$ gravitational effects become very weak inside the core of the soliton. In this limit the degenerate supermultiplet should reappear, so one expects the lowest eigenvalue to be proportional to a power of M/M_p .

In the vacuum all the supersymmetries are unbroken, and the cosmological constant vanishes. Therefore, when the supersymmetry is local, it can imply a vanishing cosmological constant without also implying undesirable degenerate bose-fermi supermultiplets.

In [2] Witten observed that the phases arising in conical geometries disturb the usual connection between supersymmetry of the vacuum and bose-fermi degeneracy of the excited states. In the present paper, this has been explicitly verified in a specific model. This model in addition exhibits an effect equal in importance to the conical phases: Aharonov-Bohm phases. Perhaps this observation might be useful for generalizing Witten's mechanism to $3 + 1$ dimensions.

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★ An exception to this occurs when $M = M_p$, for which the space is asymptotic to a cylinder and a normalizable zero mode of the form (38) exists.

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